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On the rule of mixtures for flow stresses in stainless-steel-clad aluminium sandwich sheet metals

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The flow stresses in stainless-steel-clad aluminium sandwich sheet metals followed the mixture rule which is an average of component properties weighted by volume fractions, even when transverse stresses were calculated to develop in the component layers due to their different anisotropic plastic behaviours. Such flow stresses in the sandwich sheets were attributed not to negligibly small transverse stresses compared with longitudinal stresses, but to the compensation effects of increased and decreased longitudinal stresses due to tensile and compressive transverse stresses developed in the different component layers.

1. Introduction

Stainless-steel-clad aluminium sandwich sheets have both the very good corrosion-resistant and mechanical properties of stainless steel and the excellent heat and electrical conductivities of aluminium. The flow stresses of the sandwich sheets are known to follow the mixture rule given below in Equation 1 [1, 2], just like fibre-reinforced composite materials tensioned along the fibre axis:

$$\sigma_{us} = \sigma_{uA} V_A + \sigma_{uB} V_B \qquad (1)$$

where σ_u and V indicate the uniaxial flow stress and volume fraction, and subscripts s, A and B stand for the sandwich sheet and its component A and B layers, respectively. The flow stress of a fibre-reinforced composite material may be easily understood on the basis of the isostrain hypothesis. However, the behaviour of a sandwich sheet cannot be as simple as in fibre-reinforced composite materials, because transverse stresses may develop in the component layers due to a difference in their anisotropy. Therefore, the mixture rule for the sandwich sheet has been attributed to the presence of negligibly small transverse stresses compared to axial or longitudinal stresses [2].

The purpose of this work is to examine the applicability of the mixture rule to stainless-steel-clad aluminium sandwich sheets.

2. Experimental methods

Sandwich sheets of (304 stainless steel)–aluminium–(304 stainless steel) of 2 to 3 mm thickness were fabricated by rolling at 400 to 500° C, during which the stainless steel sheets were reduced by 4 to 10% and the commercial-purity aluminium sheets by 30 to 48% (Table I). The clad sheets were subsequently annealed at 400° C for 15 min to remove residual stresses in the sheets and to improve the bond strength between the layers. The oxide scale on the sheet surface was removed by 10% nitric acid at 70° C.

Tensile specimens of 50 mm gauge length were elongated at a crosshead speed of $10\,\mathrm{mm\,min^{-1}}$ to obtain flow curves. Tensile specimens of stainless steel were cut from both stainless steel sheets obtained by dissolving the aluminium of the sandwich sheets in sodium hydroxide solution, and from stainless steel sheets as received. Aluminium sheets for tensile tests were fabricated by rolling at the same reduction and temperature as in the sandwich sheet fabrication.

The plastic strain ratios, R, were measured at an engineering strain of 0.15 in accordance with ASTM E517-74. The dimensional changes of an R specimen were obtained by measuring the dimensions of a square grid photoprinted on the specimen surface within 0.001 mm.

All the properties mentioned above were measured at 0, 45 and 90° to the rolling direction and averaged using the equation

$$\bar{X} = (X_0 + 2X_{45} + X_{90})/4$$

where X_0, X_{45} and X_{90} are the properties at 0, 45 and 90° to the rolling direction.

The martensite transformation takes place in 304 stainless steel as it deforms plastically. The transformation is known to be negligible at 15% tensile strain [3, 4], and this was also confirmed in the present work. Therefore it was not necessary to consider the volume change due to the martensite transformation in the R value measurements.

3. Results and discussion

A typical example of flow curves of the sandwich sheets is shown in Fig. 1. The flow curves are shown to follow the rule of mixture of Equation 1. When transverse stresses develop in the component layers of the sheet due to their different anisotropic plastic behaviours, the flow stress of the sandwich sheet should be expressed as

$$\sigma_{us} = \sigma_{1A} V_A + \sigma_{1B} V_B \qquad (2)$$

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TABLE I Stainless-steel-clad aluminium sandwich sheet metal fabrication conditions*

Sample No.	Initial thickness (mm)		Total reduction ratio (%)	Final thickness (mm)	Composition (vol %)		Reduction ratio (%)	
	SLS	Al			SLS	Al	SLS	Al
Sandwich sheet metal A (rolling temperature: 400° C)								
A-1	0.4 × 2	2.0	28.6	2.0	0.38	0.62	9.6	36
A-2	0.5 × 2	2.0	33.3	2.0	0.48	0.52	6.2	46
A-3	0.4 × 2	2.5	30.3	2.3	0.33	0.67	10.5	36
A-4	0.5 × 2	2.5	28.6	2.5	0.38	0.62	9.8	35
A-5	0.4 × 2	3.0	31.6	2.6	0.28	0.72	14.0	36
A-6	0.5 × 2	3.0	25	3.0	0.31	0.69	7.6	30
Sandwich sheet metal B (rolling temperature: 450° C)								
B-1	0.4 × 2	2.0	28.6	2.0	0.40	0.60	7.0	37
B-2	0.5 × 2	2.0	33.3	2.0	0.50	0.50	6.0	47
B-3	0.4 × 2	2.5	30.3	2.3	0.34	0.66	5.8	38
B-4	0.5 × 2	2.5	28.6	2.5	0.41	0.59	5.2	38
B-5	0.4 × 2	3.0	31.6	2.6	0.31	0.69	8.6	39
B-6	0.5 × 2	3.0	25	3.0	0.33	0.67	5.9	32
Sandwich sheet metal C (rolling temperature: 500° C)								
C-1	0.4 × 2	2.0	28.6	2.0	0.40	0.60	4.4	39
C-2	0.5 × 2	2.0	33.3	2.0	0.51	0.49	4.2	48
C-3	0.4 × 2	2.5	30.3	2.3	0.36	0.64	5.0	39
C-4	0.5 × 2	2.5	28.6	2.5	0.41	0.59	4.5	39
C-5	0.4 × 2	3.0	31.6	2.6	0.31	0.69	5.0	39
C-6	0.5 × 2	3.0	25	3.0	0.33	0.67	4.8	33

*SLS: stainless steel, Al: aluminium.

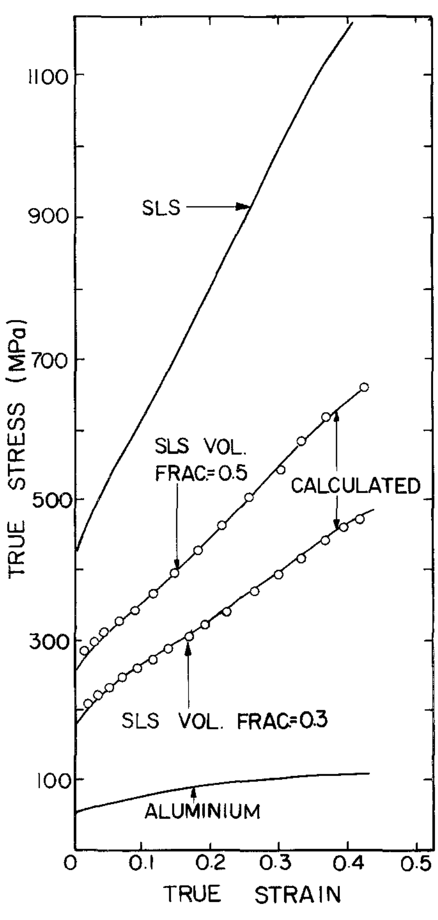

Figure 1 Flow curves of stainless steel (SLS), aluminium and sandwich sheets. The stainless steel and aluminium specimens were worked similarly to the layers in composite sheets. (○) Measured values.

while the stresses in the transverse and thickness directions in the component layers A and B are related by

$$\sigma_{2s} = 0 = \sigma_{2A} V_A + \sigma_{2B} V_B \quad (3)$$

and

$$\sigma_{3A} = \sigma_{3B} = 0 \quad (4)$$

Here σ_1, σ_2 and σ_3 are the stresses in the tensile, transverse and thickness directions, respectively (Fig. 2).

The stresses σ_{1A}, σ_{1B}, σ_{2A} and σ_{2B} can be calculated using an appropriate yield criterion. Semiatin [2] used Hill's quadratic yield criterion for anisotropic materials:

$$F|\sigma_2 - \sigma_3|^2 + G|\sigma_3 - \sigma_1|^2 + H|\sigma_1 - \sigma_2|^2 = 1 \quad (5)$$

where F, G and H are constants which characterize the anisotropy. Recently the Hill criterion has been found to be inappropriate. Some new yield criteria have been

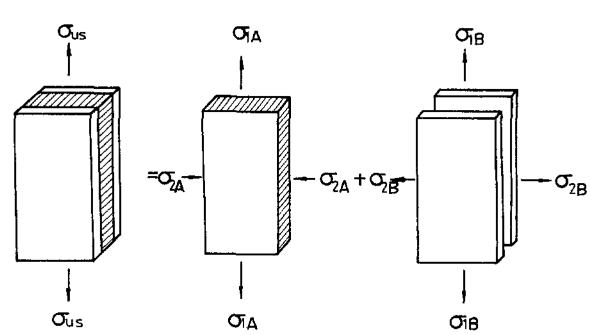

Figure 2 Isostrain model for uniaxial tensile deformation of clad sheet metal. See Equations 2, 3 and 4.

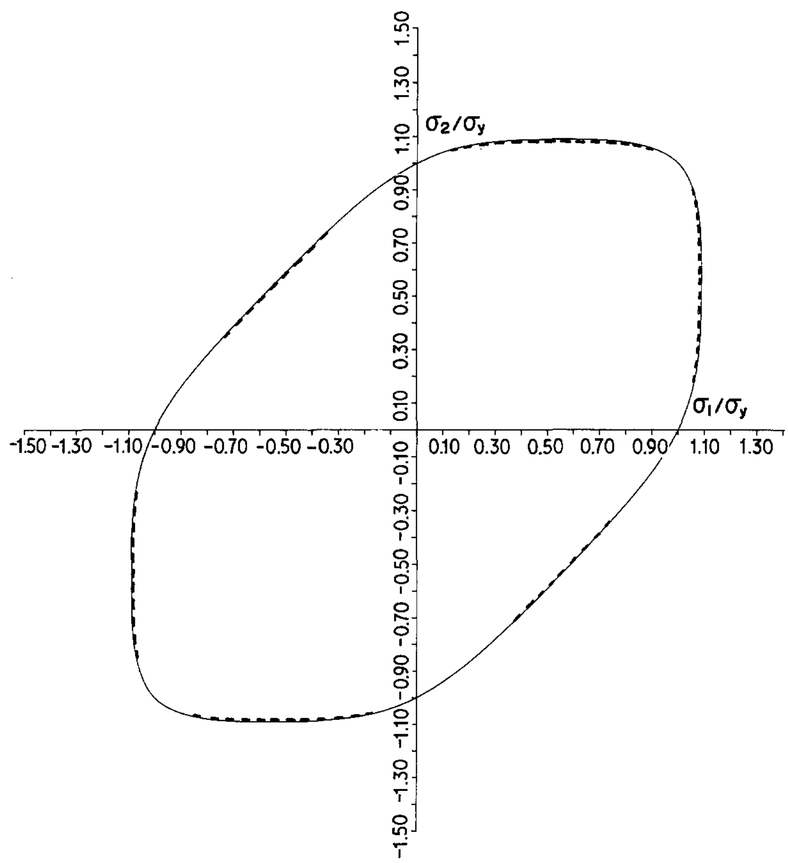

Figure 3 Yield loci for isotropic f c c metals. The solid line yield locus was calculated by setting $a = 8$ in Equation 7. The dashed line yield locus was calculated by the Bunge method based on Taylor's minimum energy theory.

advanced to predict the yielding of anisotropic materials.

Hosford [5] has proposed a modification to Equation 1 which can be expressed as

$$F|\sigma_2 - \sigma_3|^a + G|\sigma_3 - \sigma_1|^a + H|\sigma_1 - \sigma_2|^a = 1 \quad (6)$$

where $a = 6$ for b c c metals and $a = 8$ to 10 for f c c metals. For planar isotropic materials in the plane stress state ($\sigma_{33} = 0$), Equation 6 reduces to the equation

$$|\sigma_1|^a + |\sigma_2|^a + R|\sigma_1 - \sigma_2|^a = (R + 1)\sigma_y^a \quad (7)$$

where R is the plastic strain ratio and σ_y is the uniaxial yield stress along the plane direction. The yield criterion fits very well with the yield loci calculated based on Taylor's minimum energy theory [6] for the deformation of a crystalline body. Figs 3 and 4 show yield loci for f c c and b c c metals whose grains are randomly distributed, calculated by the Bunge method [7] which is based on the Taylor theory. In the calculation the f c c and b c c metals were assumed to have the $\{111\}\langle 110\rangle$ and $\{hkl\}\langle 111\rangle$ slip systems, respectively. The yield loci in Figs 3 and 4 can be best fitted by Equation 7 with $a = 8$ and 6, respectively, for the value $R = 1$ which applies to isotropic materials. Fig. 5 shows the yield loci calculated on the basis of the Taylor theory for metals with the $\{111\}\langle 110\rangle$ or $\{110\}\langle 111\rangle$ slip systems and with strain ratios of 1.93 and 0.59. The yield loci in Fig. 5 can be very well represented by Equation 7 with $a = 8$. Lee [8, 9] derived a theoretical relation between the limiting drawing ratio and the plastic strain ratio using Equation 7. The relation agreed very well with the measured relation at $a = 8$ for cubic-system metals. The above examples suggest that the exponent a in Equation 7 need not vary with the degree of anisotropy which is reflected by the plastic strain ratio.

Other attempts at modifying Equation 6 have been made by Hill [10], Bassani [11] and Gotoh [12]. Hill's new yield criterion for plane stress condition can be expressed as

$$(1 + 2R)|\sigma_1 - \sigma_2|^m + |\sigma_1 + \sigma_2|^m = 2(1 + R)\sigma_y^m \quad (8)$$

where $m > 1$. The exponent m is empirically expressed [13] as

$$m = 1.14 + 0.86R \quad \text{for} \quad R < 1$$
$$m = 2 \quad \text{for} \quad R > 1 \quad (9)$$

We are still not sure which criterion is the best. In this paper Equations 7 and 8 will be used as yield criteria for planar isotropic materials in the plane stress condition. It is noted that setting $a = 2$ in Equation 7 and $m = 2$ in Equation 8 results in Hill's quadratic yield criterion under the plane stress condition.

Strain components can be obtained from an appropriate stress function using the associated flow rule

$$d\varepsilon_{ij} = d\lambda \frac{\partial f(\sigma_{ij})}{\partial \sigma_{ij}} \quad (10)$$

where $d\varepsilon_{ij}$ and $d\lambda$ are the strain increment and a proportionality factor, respectively. For planar isotropic materials in the plane stress condition, the stress function may be, from Equations 7 and 8, expressed as

$$f_1(\sigma_{ij}) = |\sigma_1|^a + |\sigma_2|^a + R|\sigma_1 - \sigma_2|^a \quad (11)$$

$$f_2(\sigma_{ij}) = (1 + 2R)|\sigma_1 - \sigma_2|^m + |\sigma_1 + \sigma_2|^m \quad (12)$$

It follows from Equations 10 and 11, and $d\varepsilon_1$ +

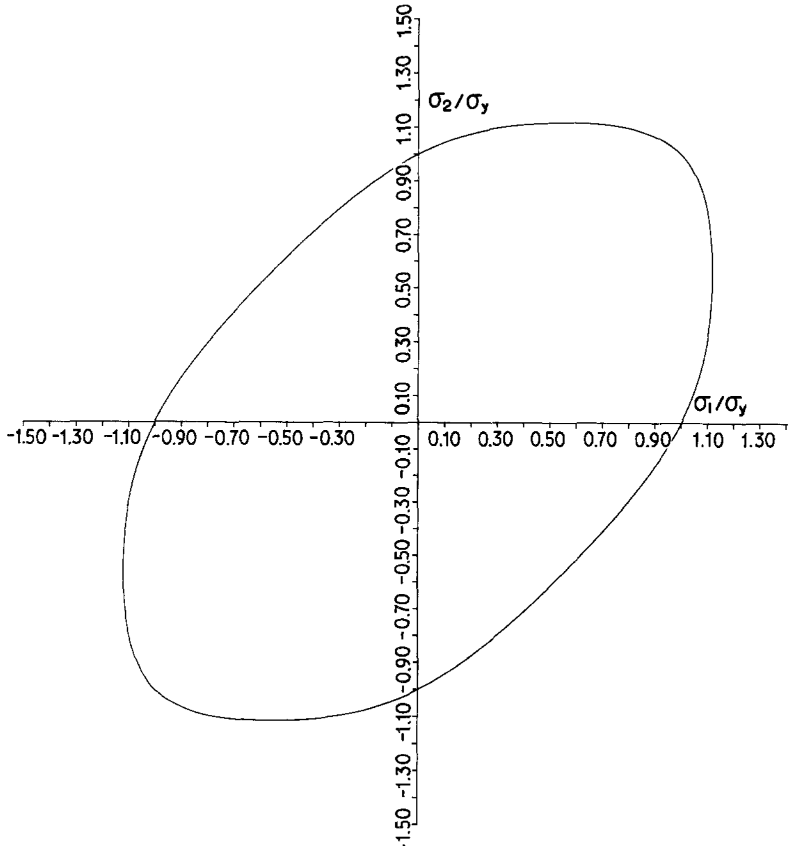

Figure 4 Yield locus for isotropic bcc metals. The locus was calculated by setting $a = 6$ in Equation 7, which is practically identical to that calculated by the Bunge method on the basis of Taylor's minimum energy theory.

$d\varepsilon_2 + d\varepsilon_3 = 0$, that

$$R = \frac{d\varepsilon_2}{d\varepsilon_3} = \left(\frac{|\sigma_2|^a}{\sigma_2} - R\frac{|\sigma_1 - \sigma_2|^a}{(\sigma_1 - \sigma_2)}\right) \Big/ \left(-\frac{|\sigma_1|^a}{\sigma_1} - \frac{|\sigma_2|^a}{\sigma_2}\right) \qquad (13)$$

It follows from Equations 10 and 12 that

$$R = \frac{d\varepsilon_2}{d\varepsilon_3} = \frac{-(1 + 2R)|\sigma_1 - \sigma_2|^m[(\sigma_1 + \sigma_2)/(\sigma_1 - \sigma_2)] + |\sigma_1 + \sigma_2|^m}{-2|\sigma_1 + \sigma_2|^m} \qquad (14)$$

Since the component layers of the sandwich sheet and the sandwich sheet itself are subjected to an equal strain, their plastic strain ratios should be the same, that is

$$R_s = \frac{d\varepsilon_{2s}}{d\varepsilon_{3s}} = \frac{d\varepsilon_{2A}}{d\varepsilon_{3A}} = \frac{d\varepsilon_{2B}}{d\varepsilon_{3B}}$$

Therefore it follows from Equation 13 that

$$R_s = \frac{d\varepsilon_{2s}}{d\varepsilon_{3s}} = \frac{-(|\sigma_{2A}|^a/\sigma_{2A}) + [R_A|\sigma_{1A} - \sigma_{2A}|^a/(\sigma_{1A} - \sigma_{2A})]}{(|\sigma_{1A}|^a/\sigma_{1A}) + (|\sigma_{2A}|^a/\sigma_{2A})}$$

$$= \frac{-(|\sigma_{2B}|^a/\sigma_{2B}) + [R_B|\sigma_{1B} - \sigma_{2B}|^a/(\sigma_{1B} - \sigma_{2B})]}{(|\sigma_{1B}|^a/\sigma_{1B}) + (|\sigma_{2B}|^a/\sigma_{2B})} \qquad (15)$$

From Equation 14 it follows that

$$R_s = \frac{-(1 + 2R_A)|\sigma_{1A} - \sigma_{2A}|^m[(\sigma_{1A} + \sigma_{2A})/(\sigma_{1A} - \sigma_{2A})] + |\sigma_{1A} + \sigma_{2A}|^m}{-2|\sigma_{1A} + \sigma_{2A}|^m}$$

$$= \frac{-(1 + 2R_B)|\sigma_{1B} - \sigma_{2B}|^m[(\sigma_{1B} + \sigma_{2B})/(\sigma_{1B} - \sigma_{2B})] + |\sigma_{1B} + \sigma_{2B}|^m}{-2|\sigma_{1B} + \sigma_{2B}|^m} \qquad (16)$$

The yield conditions of the layers A and B in the sandwich sheet can be expressed, from Equation 7, as

$$|\sigma_{1A}|^a + |\sigma_{2A}|^a + R_A|\sigma_{1A} - \sigma_{2A}|^a = (R_A + 1)\sigma_{uA}^a \qquad (17a)$$

$$|\sigma_{1B}|^a + |\sigma_{2B}|^a + R_B|\sigma_{1B} + \sigma_{2B}|^a = (R_B + 1)\sigma_{uB}^a \qquad (17b)$$

and, from Equation 8,

$$(1 + 2R_A)|\sigma_{1A} - \sigma_{2A}|^m + |\sigma_{1A} + \sigma_{2A}|^m = 2(1 + R_A)\sigma_{uA}^m \qquad (18a)$$

$$(1 + 2R_B)|\sigma_{1B} - \sigma_{2B}|^m + |\sigma_{1B} + \sigma_{2B}|^m = 2(1 + R_B)\sigma_{uB}^m \qquad (18b)$$

In the above equations the flow stress σ_u was substituted for the yield stress σ_y, to consider strain-hardening materials.

The values of σ_{1A}, σ_{1B}, σ_{2A} and σ_{2B} can be calculated using Equations 3, 15 and 17, or Equations 3, 16 and 18, into which the measured values of V_A, V_B, R_A, R_B, σ_{uA} and σ_{uB} are substituted.

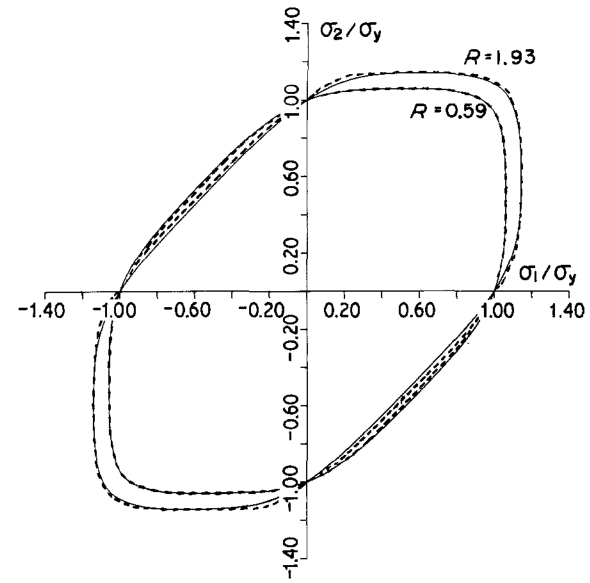

Figure 5 Yield loci for fcc metals with $R = 0.59$ and $R = 1.93$, calculated on the basis of Taylor's minimum energy theory (dashed curves), compared with those calculated by setting $a = 8$ in Equation 7 (solid curves).

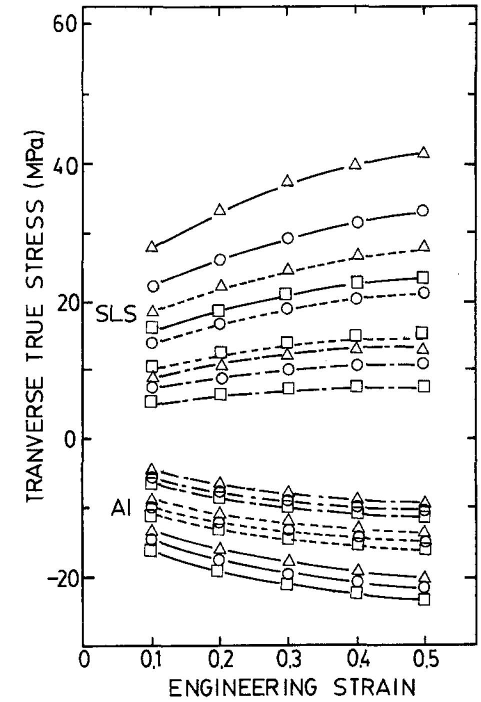

Figure 6 Transverse stresses in stainless steel (SLS) and aluminium layers calculated on the basis of various yield criteria. (–·–) $a = 8$ in Equations 3, 15 and 17; (– – –) $a = 2$ in Equations 3, 15 and 17; (——) Equations 3, 16 and 18. Volume fraction of SLS (△) 0.33, (○) 0.40, (□) 0.50.

The flow stresses of aluminium and stainless steel comprised in a sandwich sheet fabricated at 500° C are given in Table II.

The transverse stresses, σ_{2A} and σ_{2B}, and the ratio of the transverse stress to the longitudinal stress, σ_2/σ_1, calculated using the data in Table II are shown in Figs 6 and 7. These indicate that different yield criteria yield different transverse and longitudinal stresses, and the ratio of the transverse stress to the longitudinal stress can even exceed 0.2. Therefore we are not confident that the transverse stress can be neglected in the calculation of flow stresses in sandwich sheets.

The flow stresses calculated using the Rule of Mixtures (Equation 1 and Equation 2) are given in Table III. The stresses calculated using Equation 1 are practically equivalent to the measured flow stresses. Equation 2 can give rise to different stresses depending on the yield criteria used to calculate the longitudinal stresses. The flow stresses calculated on the basis of the Hosford yield criterion are in better agreement with those calculated by the mixture rule than those based on the two other yield criteria. However, differences between the flow stresses calculated on the basis of the different yield criteria are practically negligible. Therefore the fact that the Rule of Mixtures satisfies the flow stresses in sandwich sheets cannot be explained by Semiatin's argument [2] that the transverse stresses are negligible compared with the longitudinal stresses.

For the convenience of explanation the yield criteria, Equations 7 and 8, may be approximated by

$$\sigma_y = \sigma_1 - \alpha\sigma_2 \quad \text{for} \quad |\sigma_2| < 0.2|\sigma_1| \quad (19)$$

where α is defined by the reciprocal of the slope of the tangent at $\sigma_1 = \sigma_y$ and $\sigma_2 = 0$ of yield loci. Therefore α is expressed as

$$\alpha = \left(\frac{d\sigma_1}{d\sigma_2}\right)_{\sigma_1=\sigma_y,\sigma_2=0} = \frac{R}{1+R} \quad (20)$$

regardless of the yield criteria, Equations 7 and 8. It follows from Equation 19 that

$$\sigma_{uA} = \sigma_{1A} - \alpha_A\sigma_{2A} \quad (21a)$$

$$\sigma_{uB} = \sigma_{1B} - \alpha_B\sigma_{2B} \quad (21b)$$

where the flow stress σ_u was substituted for the yield stress σ_y in Equation 19. Substitution of Equations 3 and 21 into Equation 2 yields

$$\begin{aligned}\sigma_{us} &= \sigma_{1A}V_A + \sigma_{1B}V_B = \sigma_{uA}V_A + \sigma_{uB}V_B \\ &+ \sigma_{2B}V_B(\alpha_B - \alpha_A) = \sigma_{uA}V_A + \sigma_{uB}V_B \\ &+ \sigma_{2A}V_A(\alpha_A - \alpha_B) \quad (22)\end{aligned}$$

The quantity $(\alpha_A - \alpha_B)$ may be explicitly expressed as

$$|\alpha_A - \alpha_B| = \frac{|R_A - R_B|}{1 + R_A + R_B + R_AR_B} < 1 \quad (23)$$

The quantity $(\alpha_A - \alpha_B)$ makes the contribution of σ_{2A} or σ_{2B} less important. Therefore Equation 22 is very well approximated by the Rule of Mixtures, Equation 1.

TABLE II Flow stresses of aluminium and stainless steel sheets comprised in a sandwich sheet fabricated at 500° C*

Engineering strain (%)	10	15	20	25	30	35	40	45	50	60	$\bar{R}$
σ_{uA} (MPa)	78.4	86.3	92.2	97.1	101.0	103.9	106.9	109.8	111.8	115.7	0.4
σ_{uB} (MPa)	630.5	735.5	818.8	887.4	951.2	1004.1	1052.2	1096.3	1134.6	1206.1	0.92

*A = aluminium, B = 304 stainless steel.

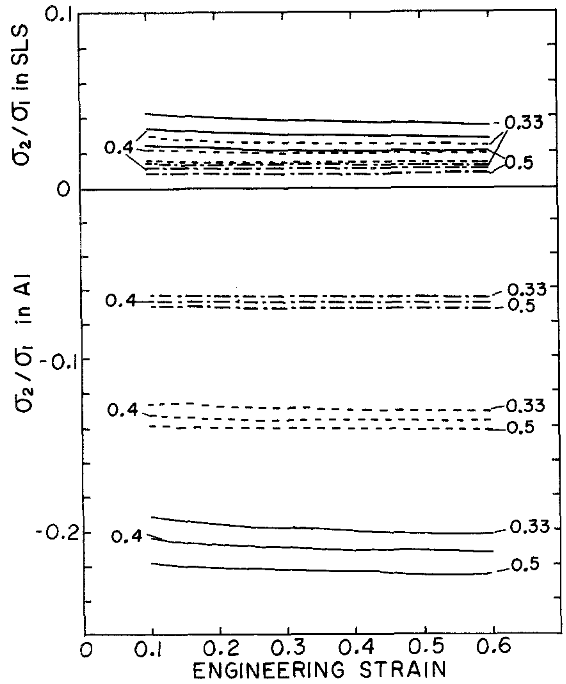

Figure 7 The ratio of transverse stress to longitudinal stress in stainless steel (SLS) and aluminium layers calculated on the basis of various yield criteria. (–·–) $a = 8$ in Equations 3, 15 and 17; (– – –) $a = 2$ in Equations 3, 15 and 17; (——) Equations 3, 16 and 18. Numerical values give the volume fraction of SLS.

Equations 15 and 16 may be used to calculate the R value or the plastic strain ratio of a sandwich sheet, since σ_1 and σ_2 in the component layers A and B can be calculated as explained earlier when the R value, flow stress and volume fraction of each component layer are known. The R values calculated on the basis of the various yield criteria are compared with the measured data in Fig. 8. The values calculated on the basis of the Hosford yield criterion ($a = 8$ in Equation 15) and Hill's new yield criterion (Equation 16) are in slightly better agreement with the measured data than the value calculated on the basis of Hill's quadratic yield criterion ($a = 2$ in Equation 15 or $m = 2$ in Equation 16).

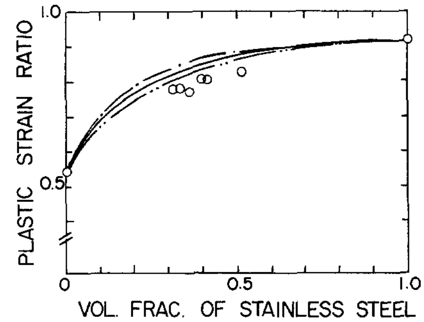

Figure 8 Plastic strain ratios of sandwich sheets as a function of the volume fraction of stainless steel. (O) Measured values; (–·–) calculated using Equation 15 with $a = 2$; (——) Equation 15 with $a = 8$; (–··–) Equation 16.

4. Conclusions

1. The longitudinal and transverse stresses developed in the component layers of stainless-steel-clad aluminium sandwich sheet metals varied substantially with the yield criteria used in the stress calculation.

2. An average of component longitudinal stresses weighted by volume fractions, which must be the theoretical flow stresses of the sandwich sheets, were almost equal to an average of component flow stresses weighted by volume fractions (the mixture rule) regardless of the yield criteria. The measured flow stresses followed the mixture rule.

3. The plastic strain ratios of the composite sheets were calculated using the plastic strain ratios, the flow stresses and the volume fractions of the component sheets based on the various yield criteria. The calculated values were generally in good agreement with the measured data, though Hosford's yield criterion and Hill's new yield criterion yielded slightly better results than Hill's quadratic yield criterion for anisotropic materials.

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TABLE III Flow stresses of sandwich sheet metals calculated on the basis of various criteria

Criterion*	Engineering strain (%)									
	10	15	20	25	30	35	40	45	50	60
Volume fraction of stainless steel = 0.33										
MIX	260.6	300.5	332.0	357.9	381.6	401.0	418.8	435.3	449.3	475.5
HIQ	260.3	301.0	332.5	358.2	382.7	401.4	419.3	435.9	449.5	476.1
HOS	260.8	300.8	332.2	358.1	381.8	401.2	418.9	435.6	449.6	475.3
HIN	261.2	301.1	332.6	358.6	382.6	401.8	419.6	436.2	450.1	479.7
Volume fraction of stainless steel = 0.4										
MIX	299.2	346.0	382.8	413.2	441.1	464.0	485.0	504.4	520.9	551.9
HIQ	299.5	346.5	383.1	413.8	441.4	464.5	485.5	504.8	521.3	552.4
HOS	299.5	346.2	383.0	413.5	441.3	464.3	485.2	504.6	521.1	552.2
HIN	300.6	346.6	383.5	414.0	441.8	464.5	485.8	505.2	521.8	552.7
Volume fraction of stainless steel = 0.5										
MIX	354.5	410.9	455.5	492.3	526.1	554.0	579.5	603.1	623.2	660.9
HIQ	354.8	411.5	456.0	492.5	526.3	554.5	580.0	603.5	623.5	661.4
HOS	354.6	411.1	455.7	492.5	526.3	554.3	579.8	603.4	623.5	661.3
HIN	355.0	411.4	456.1	492.9	526.8	554.8	580.3	603.8	623.9	661.7

*MIX: Rule of Mixtures, HIQ: Hill's quadratic yield criterion, HOS: $a = 8$ in Hosford criterion, HIN: Hill's new criterion.

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